

# Finite Supersymmetric Threshold Corrections to CKM Matrix Elements in the Large $\tan\beta$ Regime

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**Abstract:** We evaluate the finite 1-loop threshold corrections, proportional to  $\tan\beta$ , to the down quark mass matrix. These result in corrections to down quark masses and to Cabibbo-Kobayashi-Maskawa [CKM] matrix elements. The corrections to CKM matrix elements are the novel feature of this paper. For grand unified theories with large  $\tan\beta$  these corrections may significantly alter the low energy predictions of four of the CKM matrix elements and the Jarlskog parameter  $J$ , a measure of CP violation. The angles  $\alpha$ ,  $\beta$  and  $\gamma$  of the unitarity triangle and the ratio  $|\frac{V_{ub}}{V_{cb}}|$ , however, are not corrected to this order. We also discuss these corrections in the light of recent models for fermion masses. Here the corrections may be useful in selecting among the

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various models. Moreover, if one model fits the data, it will only do so for a particular range of SUSY parameters.

# 1 Introduction

Minimal supersymmetric[SUSY] grand unified theories[GUTs] based on the gauge group  $SO(10)$  require  $\tan\beta$  (the ratio of the vacuum expectation values of the two Higgs scalar doublets present in the low energy theory) to be of order  $M_{top}/M_{bottom} \approx 50$ . This follows from the unification of the top, bottom and tau Yukawa couplings at the GUT scale,  $M_{GUT}$ , and the necessity to fit the large top to bottom mass ratio at the weak scale[1]. Recent results using a general  $SO(10)$  operator analysis for fermion masses and mixing angles seem to be in significant agreement with experiment [2]. It was shown in [3, 4, 5], however, that there are potentially large finite 1-loop corrections (proportional to  $\tan\beta$ ) to the masses of the down-type quarks at the supersymmetric threshold. Note, these corrections were not included in the analysis of ref. [2]. They may be as large as several tens of per cent dependent on the sparticle spectrum[4]. Thus they must be included when analyzing any SUSY theory with large  $\tan\beta$ . In this paper we emphasize that the non-diagonal elements of the down quark mass matrix also get potentially large corrections; thus leading to significant corrections to some CKM matrix elements and the Jarlskog parameter  $J$ . Our main results are given in equations (9), (16)- (18) and (22) (or in approximate form in eqns. (25) and (28)). Note, the Cabibbo angle and the CP violating angles  $\alpha, \beta$  and  $\gamma$  are not significantly corrected to this order.

## 2 1-Loop Corrections to the Down Quark Mass Matrix

When one integrates superpartners out of the minimal supersymmetric standard model (MSSM) , there are significant  $\mathcal{O}(\tan\beta)$  1-loop corrections to the mass matrix of the down-type quarks originating in the diagrams with gluino – d-type squark and chargino – u-type squark loops yielding (see Figures 1a,b,c , for the notation and conventions used and a short derivation see Appendix)<sup>§</sup>

$$\mathbf{m}_d = (V_d^{L0})^\dagger (1 + \epsilon \mathbf{\Gamma}_d + \epsilon V_{CKM}^{0\dagger} \mathbf{\Gamma}_u V_{CKM}^0) \mathbf{m}_d^{0Diag} V_d^{R0} , \quad (1)$$

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<sup>§</sup>We would like to thank Uri Sarid for bringing our attention to the second chargino diagram , Fig.1c.

with  $\epsilon = \frac{1}{16\pi^2} \tan\beta$  and

$$\Gamma_{d\,ij} = \frac{8}{3} g_3^2 (\Gamma_{dL}^\dagger)_{i\alpha} \int dk \frac{M_{\tilde{g}}}{(k^2 + M_{\tilde{g}}^2)(k^2 + m_{\tilde{d}_\alpha}^2)} (\Gamma_{dR})_{\alpha l} \frac{(\mathbf{m}_d^{0\,Diag})_{lj}^{-1}}{\tan\beta}, \quad (2)$$

$$\begin{aligned} \Gamma_{u\,ij} = & -\lambda_{u_i}^{0\,Diag} (\Gamma_{uR}^\dagger)_{i\alpha} \int dk \frac{U_{2A}^{*\dagger} m_{\chi_A} V_{A2}}{(k^2 + m_{\chi_A}^2)(k^2 + m_{\tilde{u}_\alpha}^2)} (\Gamma_{uL})_{\alpha j} (v_u)^{-1} \\ & + g_2 (\Gamma_{uL}^\dagger)_{i\alpha} \int dk \frac{U_{2A}^{*\dagger} m_{\chi_A} V_{A1}}{(k^2 + m_{\chi_A}^2)(k^2 + m_{\tilde{u}_\alpha}^2)} (\Gamma_{uL})_{\alpha j} (v_u)^{-1}. \end{aligned} \quad (3)$$

Uncorrected mass and mixing matrices are labeled by a “0” superscript. The  $6 \times 3$  dimensional matrices  $\Gamma_{qL}$  and  $\Gamma_{qR}$  ( $q = u, d$ ) correspond to the additional transformations necessary to diagonalize squark mass matrices in a SUSY basis where quark mass matrices are diagonalized. Expressions for the  $\mathbf{\Gamma}$ ’s are rather complex since they involve the summation over the six-dimensional squark space. It has to be stressed though, that despite the explicit  $\tan\beta$  term in the denominator of (2) and a similar  $(v_u)^{-1} = (v_d \tan\beta)^{-1}$  term in (3), there will not be any actual  $\tan\beta$  suppression in the elements of the  $\mathbf{\Gamma}$ ’s. In the interaction basis (see the Feynman diagrams Fig.1a,b,c) one can easily recognize the  $R-L$  mixings among squarks in the loop and the mass insertions or mixings on the fermionic line in the diagram. Since each of these mixings and mass insertions introduces a  $\tan\beta$  unsuppressed quantity, the result represents a  $\tan\beta$  unsuppressed correction. This correction is significant since it corrects a  $\tan\beta$  suppressed mass matrix. To emphasize this fact the large Higgs vev ratio was pulled out into the  $\epsilon$ ’s in (1). As a net effect, one can expect (at least some) terms in the  $\mathbf{\Gamma}$ ’s to be of order  $(0.1 - 1)$ . These terms are then enhanced by a factor of  $\tan\beta$  (multiplying a standard small loop factor  $(16\pi^2)^{-1}$  in our definition of  $\epsilon$ ) and thus lead to significant mass matrix corrections.

Concentrating on the above mentioned diagrams one has to mention that there are also neutralino diagrams which contribute by finite  $\mathcal{O}(\tan\beta)$  terms to the  $d$  quark mass matrix. However, we have checked that (assuming degenerate gauginos at  $M_{GUT}$ ) these contributions are less than the leading gluino corrections roughly by a factor 16, as a result of smaller couplings, gaugino masses and group factors. Therefore these diagrams will not be discussed separately in this paper although they are included in our numerical analysis in section 4 where we discuss their effects.

In order to gain some intuition for the  $\mathbf{\Gamma}$ ’s one can find an explicit form for them in the following approximation. First, neglect the second chargino diagram (Fig.1c), since it is suppressed by a smaller coupling constant compared

to the diagrams in Fig.1a and Fig.1b. Then in the evaluation of the remaining two diagrams assume that squark mass matrices are diagonalized in generation space by the same rotations as the corresponding quark matrices. This approximation is valid assuming universal scalar masses and trilinear scalar interactions proportional to Yukawa interactions at the low energy SUSY scale.

That means that in this approximation the matrices  $\Gamma_{qL}$  and  $\Gamma_{qR}$ , ( $q = d, u$ ) are diagonal in generation space and are not completely trivial only because of the mixing between squarks of the same generation. The integrals in (2) and (3) are then easy to do along with the summation over  $\alpha = 1, \dots, 6$ , i.e. over the squark mass eigenstates. The  $\mathbf{\Gamma}$ 's are then proportional to the off-diagonal term of the down (up) squark mass matrix for each individual generation separately. We find

$$\mathbf{\Gamma}_{d\,ij} = \frac{8}{3} g_3^2 M_{\tilde{g}} \mu I_3(M_{\tilde{g}}^2, m_{\tilde{d}_{i1}}^2, m_{\tilde{d}_{i2}}^2) \delta_{ij} , \quad (4)$$

$$\mathbf{\Gamma}_{u\,ij} = U_{2A}^{*\dagger} m_{\chi_A} V_{A2} A_0 I_3(m_{\chi_A}^2, m_{\tilde{u}_{i1}}^2, m_{\tilde{u}_{i2}}^2) (\lambda_u^{0Diag})_{ij}^2 , \quad (5)$$

with the function  $I_3$  given by

$$I_3(a, b, c) = \frac{ab \ln(\frac{a}{b}) + bc \ln(\frac{b}{c}) + ac \ln(\frac{c}{a})}{(a-b)(b-c)(a-c)}.$$

Terms suppressed by  $\tan\beta$  have been neglected in these expressions. In this approximation both  $\mathbf{\Gamma}$  matrices are diagonal which makes calculations of the corrections to the masses and mixing angles in terms of mass eigenstates simple. Besides that, note the large hierarchy in  $\mathbf{\Gamma}_u$ , and a much milder hierarchy in  $\mathbf{\Gamma}_d$  based just on the non-equality of the squark masses. Note, if  $\mathbf{\Gamma}_d$  were completely proportional to the identity matrix (i.e. the case of complete squark degeneracy), the gluino loop would not contribute to quark mixing corrections at all <sup>¶</sup>.

One knows though, that the approximation used to derive (4) and (5) is not correct. The initial conditions at  $M_{GUT}$  need not be universal and, even if they were, squark masses and trilinear couplings run between the GUT (or

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<sup>¶</sup>Also note that the same analysis could be done for the corrections to the up quark mass matrix and the above mentioned approximation would show that the relevant  $\mathbf{\Gamma}$  matrices (analogous to (4) and (5)) become suppressed by  $\tan\beta$  after the up (instead of down) quark mass matrix is pulled out of the expression analogous to (1). Thus in this case there are no corrections proportional to  $\tan\beta$ . There are however corrections to charged lepton masses proportional to  $\tan\beta$ . These are smaller than those for down quarks but are still significant and must be included in any fermion mass analysis.

string) and the low energy SUSY scales and violate our assumptions. As a result the explicit form of the potentially significant (i.e.  $\tan\beta$  unsuppressed) elements in the  $\mathbf{\Gamma}$ 's is clouded by the fact that they no longer remain diagonal in generation space. In order to evaluate these effects we have performed a numerical analysis. The results are found in Section 4. We also show that our naive approximation, equations (4, 5), when suitably modified to take into account non-universal squark masses gives results which agree to within 25% with the two-loop numerical analysis.

In order to figure out the explicit form of the 1-loop threshold corrections to the CKM matrix elements as well as to quark masses in terms of the  $\mathbf{\Gamma}$  matrix elements one can define an unknown hermitian matrix  $\mathbf{B}$  as

$$V_d^L = (1 + i\epsilon\mathbf{B}) V_d^{L0} \quad (6)$$

where, again,  $V_d^{L0}$  is the matrix diagonalizing down quarks in the absence of the SUSY corrections. Since there are no large ( i.e.  $\mathcal{O}(\tan\beta)$  ) corrections to the up quark mass matrix

$$V_{CKM} \equiv V_u^L V_d^{L\dagger} = V_u^{L0} (V_d^{L0})^\dagger (1 - i\epsilon\mathbf{B}) = V_{CKM}^0 (1 - i\epsilon\mathbf{B}) . \quad (7)$$

$\mathbf{B}$  is determined through the diagonalization condition

$$(\mathbf{m}_d^{\text{Diag}})^2 \equiv \text{Diag}(m_{d_1}^2, m_{d_2}^2, m_{d_3}^2) = V_d^L \mathbf{m}_d \mathbf{m}_d^\dagger V_d^{L\dagger} , \quad (8)$$

where both  $V_d^L$  and  $\mathbf{m}_d$  on the r.h.s. are to be expanded to first order in  $\epsilon$  according to (6) and (1).

## 2.1 Corrections to Down Quark Masses

Diagonal elements of this matrix equation (8) specify the corrections to the masses of the  $d$ ,  $s$  and  $b$  ( $d_1$ ,  $d_2$  and  $d_3$ ) quarks. Note that the terms containing unknown  $\mathbf{B}$  elements drop out of these equations :

$$\frac{\delta m_{d_i}}{m_{d_i}} = \epsilon \text{Re}(\mathbf{\Gamma}_d)_{ii} + \epsilon [V_{CKM}^{0\dagger} \text{Re}(\mathbf{\Gamma}_u) V_{CKM}^0]_{ii} . \quad (9)$$

This is an exact formula where the effects of squark rotations are fully included in the  $\mathbf{\Gamma}$ 's. Since the  $\mathbf{\Gamma}_u$  matrix has some generation hierarchy (for more discussion on this see Section 4) due to the Yukawa couplings in the chargino

loop the dominant correction from the chargino diagram goes to the  $b$  quark mass correction:

$$\left(\frac{\delta m_b}{m_b}\right)_{\chi^+} = \epsilon \operatorname{Re}(\mathbf{\Gamma}_u)_{33} + \epsilon \mathcal{O}(10^{-3}) . \quad (10)$$

The suppression in the second term above is caused by the hierarchies present in (9). The largest next-to-leading correction indicated above results, for example, from the term  $(V_{CKM}^{0\dagger})_{32} \operatorname{Re}(\mathbf{\Gamma}_u)_{23} (V_{CKM}^0)_{33}$  where two orders come from  $V_{cb}^*$  and at least one order from  $\mathbf{\Gamma}_{u32}$ .

Note that the corrections to the masses of the  $s$  and  $d$  quarks can easily be as significant, or even larger than the correction to the  $b$  quark mass. While the gluino correction (which is the largest correction to each quark mass) to the  $b$  quark mass is larger due to the smaller  $b$  squark masses (in a universal-like scenario where one starts with all soft squark masses equal at the GUT scale), the chargino correction may invert the net effect since it is always of opposite sign to the gluino correction and its contribution to the two lighter quarks is small.

## 2.2 Corrections to CKM Matrix Elements

The non-diagonal equations, i.e. those with zeros on the l.h.s. of the matrix equation (8), lead to

$$-i\epsilon \mathbf{B}_{ij} = \epsilon \mathbf{\Gamma}_{d\,ij} + \epsilon (V_{CKM}^{0\dagger} \mathbf{\Gamma}_u V_{CKM}^0)_{ij} (1 + \mathcal{O}(\frac{m_{d_i}^2}{m_{d_j}^2})) , \quad (11)$$

where  $ij$  indices correspond to the 12, 13 or 23 combinations and the transposed elements (for  $i > j$ ) are obtained by the hermiticity of  $\mathbf{B}$ . The diagonal elements of  $\mathbf{B}$  remain undetermined by this procedure but to the first order in the  $\epsilon$ -expansion they can be removed by phase redefinitions of the  $b$ ,  $s$  and  $d$  fields. We thus set the diagonal elements of  $\mathbf{B}$  to zero.

Then from (7) we can easily derive <sup>||</sup>

$$\begin{aligned} \delta V_{cb} &= \epsilon [V_{cd} \mathbf{\Gamma}_{d\,13} + V_{cs} \mathbf{\Gamma}_{d\,23}] \\ &\quad + \epsilon [\{\delta_{2j} - (V_{CKM})_{23} (V_{CKM}^\dagger)_{3j}\} \mathbf{\Gamma}_{u\,jk} (V_{CKM})_{k3}] \\ \delta V_{ub} &= \epsilon [V_{ud} \mathbf{\Gamma}_{d\,13} + V_{us} \mathbf{\Gamma}_{d\,23}] \end{aligned} \quad (12)$$

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<sup>||</sup>Zero superscripts are dropped from now on since they make no difference in the following expressions.

$$+ \epsilon [ \{ \delta_{1j} - (V_{CKM})_{13} (V_{CKM}^\dagger)_{3j} \} \mathbf{\Gamma}_{u_{jk}} (V_{CKM})_{k3} ] \quad (13)$$

$$\begin{aligned} \delta V_{td} = & -\epsilon [ V_{ts} \mathbf{\Gamma}_{d_{21}}^\dagger + V_{tb} \mathbf{\Gamma}_{d_{31}}^\dagger ] \\ & - \epsilon [ \{ \delta_{3j} - (V_{CKM})_{31} (V_{CKM}^\dagger)_{1j} \} \mathbf{\Gamma}_{u_{jk}}^\dagger (V_{CKM})_{k1} ] \end{aligned} \quad (14)$$

$$\begin{aligned} \delta V_{ts} = & \epsilon [ V_{td} \mathbf{\Gamma}_{d_{12}} - V_{tb} \mathbf{\Gamma}_{d_{32}}^\dagger ] + \epsilon [ (V_{CKM})_{31} (V_{CKM}^\dagger)_{1j} \mathbf{\Gamma}_{u_{jk}} \\ & - (V_{CKM})_{33} (V_{CKM}^\dagger)_{3j} \mathbf{\Gamma}_{u_{jk}}^\dagger ] (V_{CKM})_{k2} . \end{aligned} \quad (15)$$

As already mentioned, the numerical analysis shows that both  $\mathbf{\Gamma}_u$  and  $\mathbf{\Gamma}_d$  matrices keep track of the generation hierarchy from the Yukawa sector with the 33 element of the order of 0.1-1 and the relevant 12, 13 and 23 elements of small magnitude. Together with the hierarchy in the CKM matrix this implies that in each of the previous equations the dominant correction is the one containing the  $\mathbf{\Gamma}_{u_{33}}$  term. However the corrections due to the other terms are non-negligible resulting in a 25% effect. Thus a good approximation to the exact results of equations (12) - (15) is given by (for more detail, see Section 4)

$$\frac{\delta V_{cb}}{V_{cb}} \approx -\epsilon [ \mathbf{\Gamma}_{u_{33}} - \mathbf{\Gamma}_{u_{22}} - \frac{V_{cs}}{V_{cb}} \mathbf{\Gamma}_{d_{23}} ] \equiv -\epsilon \Delta \quad (16)$$

$$\frac{\delta V_{cb}}{V_{cb}} \approx \frac{\delta V_{ub}}{V_{ub}} \approx -\epsilon \Delta \quad (17)$$

$$\frac{\delta V_{ts}}{V_{ts}} \approx \frac{\delta V_{td}}{V_{td}} \approx -\epsilon \Delta^* . \quad (18)$$

The results of equations (17) and (18) follow directly from the unitarity of the CKM matrix and the fact that these are the only terms which receive significant corrections.

*Note that as a consequence the ratio  $V_{ub}/V_{cb}$  remains unchanged.* In addition the numerical analysis shows that  $\text{Re}\mathbf{\Gamma}_{u_{33}} \gg \text{Im}\mathbf{\Gamma}_{u_{33}}$  thus these dominant corrections to the CKM elements are equal in magnitude, but opposite in sign, to the chargino corrections to the  $b$  quark mass, eq.(10).

The other five CKM elements get the corrections of the form similar to (12) - (15). However, large  $\mathbf{\Gamma}$  elements are always in the product with a small CKM matrix element, and the terms containing large diagonal CKM matrix elements are in the same way pushed down by small  $\mathbf{\Gamma}$  elements in these corrections. Hence the actual numerical values of the corrections to  $V_{ud}$ ,  $V_{us}$ ,  $V_{cd}$ ,  $V_{cs}$  and  $V_{tb}$  are not significant, at least not at the present level of experimental accuracy. As an example, the dominant correction to, let's say  $V_{us}$  goes like

$$\delta V_{us} \sim \epsilon V_{ub} V_{ts} V_{tb}^* \mathbf{\Gamma}_{u_{33}}^* < 0.001 . \quad (19)$$



### 3 CP Violating Parameters

The Jarlskog parameter which measures CP violation can be obtained from the four CKM-matrix elements left after crossing out any row and any column of this matrix[8]:

$$J \sum_{\gamma,l} \epsilon_{\alpha\beta\gamma} \epsilon_{jkl} = \text{Im} [V_{\alpha j} V_{\beta k} V_{\alpha k}^* V_{\beta j}^*] . \quad (20)$$

Consider the product

$$J = \text{Im} [V_{cs} V_{tb} V_{cb}^* V_{ts}^*] \quad (21)$$

Using the formula (16) and (18) from the previous section it is easy to obtain the leading correction

$$\delta J \approx -2\epsilon \text{Re}[\Delta] J \quad (22)$$

*This threshold correction to J may significantly alter the prediction for  $\epsilon_K$  in SUSY GUT models with large  $\tan\beta$ .*

Note, it is not obvious how this result is obtained for other equivalent definitions of J. For example, at first glance one might guess that  $\delta J \approx 0$  for J defined by  $J = \text{Im} [V_{ud} V_{cs} V_{us}^* V_{cd}^*]$ . However, such a guess does not take into account that we have the imaginary part of the product in (20) and imaginary parts are small for every CKM matrix element, even if its absolute value is close to one. In this case the small corrections to the large CKM matrix elements become important and, in fact, it is corrections to  $V_{cs}$  and  $V_{us}$  that lead to the result (22).

Finally, we note that although J changes, *the angles of the unitarity triangle remain uncorrected to this order*. This is easily understood from a geometrical point of view. For the “standard” choice of its sides –  $|V_{ud} V_{ub}^*|$ ,  $|V_{cd} V_{cb}^*|$  and  $|V_{td} V_{tb}^*|$  – each side contains one element which gets a significant correction and (as a consequence of the unitarity of the CKM matrix discussed earlier) these corrections are identical in magnitude (see (16) and (18)). Hence, the sides are contracted (or stretched) by the same multiplicative factor and the angles stay the same. The area of the triangle gets corrected, of course, twice as much as the sides, and that is the reason for the factor of two in (22) (recall that J measures the area of the triangle).

### 4 Numerical Analysis and Conclusions

In our numerical analysis we took the initial conditions (values at the GUT scale) for the dimensionless couplings from the SO(10) models [2] which give

predictions for the low energy data in good agreement with experiment. The initial values for the dimensionful soft SUSY breaking parameters were taken from ref.[4, 12] in order to guarantee the radiative electroweak symmetry breaking at the weak scale. We focused mainly on simple non-universal cases. The numerical results presented below were obtained for  $m_{H_1}^2 = 2.0m_0^2$ ,  $m_{H_2}^2 = 1.5m_0^2$  and all other scalar masses equal  $m_0^2$ . Next, we used 2-loop renormalization group equations[10] to run all the couplings and mass parameters to the low energy scale. Leading corrections to the CKM matrix elements have appeared practically independent of the exact value of the low energy SUSY scale between  $M_Z$  and 500 GeV (changes were within 1% of the mass or the CKM element in question). In the actual numerical analysis the  $\Gamma$ -matrices have been evaluated according to the following formulae (note that there are no divergent pieces from the integrals in (2) and (3) and that the chargino summation is easy to do) :

$$\Gamma_{d_{ij}} = \frac{8}{3}g_3^2 (\Gamma_{dL}^\dagger)_{i\alpha} \frac{-m_{\tilde{d}_\alpha}^2}{m_{\tilde{d}_\alpha}^2 - M_{\tilde{g}}^2} \ln \frac{m_{\tilde{d}_\alpha}^2}{M_{\tilde{g}}^2} (\Gamma_{dR})_{\alpha j} \frac{M_{\tilde{g}}}{m_{d_j} \tan\beta} \quad (23)$$

$$\begin{aligned} \Gamma_{u_{ij}} = & \lambda_{u_i} (\Gamma_{uR}^\dagger)_{i\alpha} (m_{\tilde{u}_\alpha}^2 - |M_2|^2) I_3(m_{\chi_1}^2, m_{\chi_2}^2, m_{\tilde{u}_\alpha}^2) (\Gamma_{uL})_{\alpha j} \frac{\mu}{v_u} \\ & - g_2^2 (\Gamma_{dL}^\dagger)_{i\alpha} I_3(m_{\chi_1}^2, m_{\chi_2}^2, m_{\tilde{u}_\alpha}^2) (\Gamma_{uL})_{\alpha j} M_2 \mu. \end{aligned} \quad (24)$$

$M_2$  is the wino mass parameter and terms suppressed by  $\tan\beta$  were dropped. The summation is only over  $\alpha = 1, \dots, 6$  (there's no summation over  $i, j$  on the r.h.s. of these equations). This summation could be done analytically in terms of the mass eigenvalues, however the expressions are long and don't provide much insight, so we keep rather the compact forms above.

Typical values for these matrices at the weak scale follow -

$$\begin{aligned} \epsilon \Gamma_d & \approx \begin{pmatrix} 0.180 + i 3 \cdot 10^{-8} & -3 \cdot 10^{-6} - i 9 \cdot 10^{-7} & 8 \cdot 10^{-5} + i 2 \cdot 10^{-5} \\ -3 \cdot 10^{-6} + i 7 \cdot 10^{-7} & 0.180 - i 4 \cdot 10^{-8} & -4 \cdot 10^{-4} + i 6 \cdot 10^{-6} \\ -9 \cdot 10^{-5} + i 8 \cdot 10^{-5} & 3 \cdot 10^{-5} + i 1 \cdot 10^{-5} & 0.218 - i 3 \cdot 10^{-9} \end{pmatrix} \\ \epsilon \Gamma_u & \approx \begin{pmatrix} -0.022 + i 5 \cdot 10^{-16} & -9 \cdot 10^{-7} + i 3 \cdot 10^{-7} & -3 \cdot 10^{-6} + i 6 \cdot 10^{-6} \\ -9 \cdot 10^{-7} - i 3 \cdot 10^{-7} & -0.022 - i 3 \cdot 10^{-11} & -1 \cdot 10^{-4} + i 5 \cdot 10^{-9} \\ 1 \cdot 10^{-5} + i 3 \cdot 10^{-5} & 5 \cdot 10^{-4} - i 1 \cdot 10^{-8} & -0.116 + i 7 \cdot 10^{-10} \end{pmatrix} \end{aligned}$$

where in this case we used the GUT scale values,  $M_{1/2} = 400 \text{ GeV}$ ,  $m_0 = 250 \text{ GeV}$ , and  $A_0 = -1100 \text{ GeV}$ , the weak scale value,  $\mu = 270 \text{ GeV}$ , and Model 4 of [2] for the Yukawa matrices (with the weak scale values  $\lambda_t = 1.01$ ,  $\tan\beta = 53$  and  $V_{cb}^0 = 0.038$  as output). With these inputs we find  $M_{\tilde{g}} =$

1029 GeV,  $A_t = (A_u)_{33} = -(736 + i6 \cdot 10^{-7}) \text{ GeV}$ , up-squark mass eigenvalues (in GeV) (976, 976, 951, 951, 869, 695) and down-squark mass eigenvalues (in GeV) (980, 979, 949, 948, 820, 757). To gain some intuition for the size of the corrections, these particular values lead to  $\delta m_b/m_b = 10.2\%$ ,  $\delta m_s/m_s = 15.7\%$ ,  $\delta m_d/m_d = 15.7\%$ ,  $\delta V_{cb}/V_{cb} = \delta V_{ub}/V_{ub} = \delta V_{ts}/V_{ts} = \delta V_{td}/V_{td} = 8.0\%$  and  $\delta J/J = 16.6\%$ . As we discussed earlier the approximation of retaining only the  $\mathbf{\Gamma}_{u_{33}}$  term in equation(16) does not work extremely well since it predicts an 11.6% correction. However, this leading correction is then lowered by about 1.5% coming from the  $\mathbf{\Gamma}_{\mathbf{d}}$  term in (12)-(15) and by additional 2% from the subleading  $\mathbf{\Gamma}_{\mathbf{u}}$  terms.  $V_{ud}$ ,  $V_{us}$ ,  $V_{cd}$ ,  $V_{cs}$  and  $V_{tb}$  get a relative correction less than 1%, e.g.  $\delta V_{us}/V_{us} = 0.01\%$ . Similarly, the corrections to the angles  $\alpha$ ,  $\beta$  and  $\gamma$  of the unitarity triangle are much below 1%. Neutralino corrections have been included in the above numerical analysis. Their effects are as follows: the  $b$  mass is reduced by 1.6% and the masses of  $s$  and  $d$  are reduced by 1.3%. Integrating out neutralinos has less than a 1% impact on the CKM elements and CP violating parameter,  $J$ .

We would like to emphasize that such corrections are generic for a large subspace of the allowed parameter space.

## 4.1 Approximate Formulae for Mass and Mixing Angle Corrections

In eqns. (4) and (5), we presented the results of a naive approximation which assumes that squark mass matrices are diagonalized in generation space by the same rotations as the corresponding quark matrices. This approximation is valid in the case of universal scalar masses and trilinear scalar interactions proportional to Yukawa interactions when, in addition, one also neglects the renormalization group running from  $M_G$  to the low energy SUSY scale. If one now includes the effect of RG running, quark and squark mass matrices can no longer be diagonalized in generation space by the same unitary transformations and the  $A$  parameters are no longer universal. We have checked that a simple approximation for the corrections to down quark masses and the CKM matrix elements (valid to 25%) can be obtained by using the results of eqns. (4) and (5) with the values of squark and gluino masses obtained by RG running as input and by replacing  $A_0$  with  $A_t$  (for the third generation) and the chargino mass with the low energy value of  $\mu$ . This approximation has been widely used in the previous papers[3, 5, 4, 12, 13] where large bottom mass corrections have

been recognized. In particular, in this improved approximation

$$\begin{aligned}\delta m_d &\approx (\tilde{\epsilon}_{d1} + \tilde{\epsilon}_{d2} \mathcal{O}(10^{-6})) m_d \\ \delta m_s &\approx (\tilde{\epsilon}_{s1} + \tilde{\epsilon}_{s2} \mathcal{O}(10^{-4})) m_s \\ \delta m_b &\approx (\tilde{\epsilon}_{b1} + \tilde{\epsilon}_{b2} (|V_{tb}|^2 + \mathcal{O}(10^{-6}))) m_b ,\end{aligned}\tag{25}$$

where

$$\tilde{\epsilon}_{d_{i1}} = \frac{2\alpha_s}{3\pi} \mu M_{\tilde{g}} I_3(M_{\tilde{g}}^2, m_{\tilde{d}_{i1}}^2, m_{\tilde{d}_{i2}}^2) \tan\beta\tag{26}$$

$$\tilde{\epsilon}_{d_{i2}} = \frac{1}{16\pi^2} \mu A_{u_i} \lambda_{u_i}^2 I_3(\mu^2, m_{\tilde{u}_{i1}}^2, m_{\tilde{u}_{i2}}^2) \tan\beta.\tag{27}$$

The analogous corrections for CKM matrix elements, also valid to about 25%, are given by

$$\frac{\delta V_{cb}}{V_{cb}} \approx \frac{\delta V_{ub}}{V_{ub}} \approx \frac{\delta V_{ts}}{V_{ts}} \approx \frac{\delta V_{td}}{V_{td}} \approx -\tilde{\epsilon}_{b2},\tag{28}$$

where  $\tilde{\epsilon}_{b2}$  is defined in the eq.(27).

An important feature of the  $b$  quark mass correction is that the gluino and chargino contributions are of the opposite signs and thus there is a partial cancellation between them. This effect with its consequences has been carefully studied in [4, 12, 13]. In these papers it was shown that the magnitude of the gluino contribution is always two to three times larger than the chargino contribution and can be as large as 50% for universal scalar masses at  $M_G$ . For non-universal scalar masses the corrections can be smaller.

## 4.2 Consequences for Models of Fermion Masses

It is interesting to see what effect these corrections have for recent models of fermion masses and mixing angles. In the model of ref.[14] the value of  $|V_{cb}|$  is of order .054. This is large compared to the latest experimental values. In this model,  $\tan\beta$  can be either small or large. We would have to be in the large  $\tan\beta$  regime for these corrections to be significant. In addition consider models 4,6 and 9 of ref.[2]. In these models  $\tan\beta$  is expected to be large. Recall that the model independent experimental value of  $|V_{cb}|$  is  $0.040 \pm 0.003$  according to [6] or  $0.040 \pm 0.005$  based on [7]. For models 6 and 9, the predicted value of  $V_{cb} \sim .048 - .052$  is at the upper end of the experimentally allowed range. For all these models we would choose  $\mu M_{\tilde{g}} < 0$  so that the chargino correction to the  $b$  quark mass is positive and hence  $\delta V_{cb}/V_{cb} < 0$ . As a consequence the gluino

correction is negative which gives  $\delta m_b < 0$ . This has the effect of decreasing the prediction for  $m_t$ , since a smaller top Yukawa coupling is now needed to fit the experimental ratio  $m_b/m_\tau$ . These corrections apparently improve the predictions of the above models. However, the corrections to the strange and down quark masses, which are equal and negative, may be a problem since both ratios  $m_u/m_d$  and  $m_s/m_d$  were rather large and now the first one gets even bigger while the second one stays the same. This problem is exacerbated by the fact that the authors in [13] find no solutions for  $|\frac{\delta m_b}{m_b}| < 10\%$  with  $\mu M_{\tilde{g}} < 0$  consistent with both the experimental rate for  $b \rightarrow s\gamma$  and the cosmological constraint on the energy density of the Universe. There are solutions for larger values of  $|\frac{\delta m_b}{m_b}|$  but this range of parameters may seriously be constrained by the ratios  $m_u/m_d$  and  $m_s/m_d$ .

For model 4 of ref.[2] however the situation may be better. In this model  $|V_{cb}|$  is acceptably small ( $V_{cb} \sim .038 - .044$ ). However  $J$  is too small and thus the bag constant,  $B_K$ , needed to fit  $\epsilon_K$  is too large, i.e. greater than 1. In this case we need  $\delta J/J > 0$ . This would also increase  $|V_{cb}|$  by half as much, which may be acceptable. In this case the chargino correction to the  $b$  quark mass is negative. Thus the gluino correction to  $m_b$  is positive and  $\delta m_b > 0$ . As a result the top quark mass prediction increases. This restricts the magnitude of the effect to values of  $|\frac{\delta m_b}{m_b}| < 10\%$ . In this case both  $m_s$  and  $m_d$  increase, which improves the agreement with experiment in the  $m_s/m_d - m_u/m_d$  plane. Finally the  $b \rightarrow s\gamma$  decay rate and the cosmological constraint can be satisfied[13].

Note that in either scenario the angles  $\alpha$ ,  $\beta$  and  $\gamma$  of the unitarity triangle and the ratio  $V_{ub}/V_{cb}$  remain unchanged. These correlations of quark mass and mixing angle predictions with the sign of  $\mu M_{\tilde{g}}$  are very intriguing, especially since this sign may be determined independently once SUSY particles are observed. In a particular model the allowed maximal corrections to masses and mixing angles may represent new constraints on the magnitude and sign of the SUSY parameters.

In summary, finite SUSY corrections to the masses of the down-type quarks may be significant in the limit of large  $\tan\beta$ . In this paper we have shown that the CKM matrix elements  $V_{cb}$ ,  $V_{ub}$ ,  $V_{ts}$  and  $V_{td}$  receive similar corrections, while the correction to the Jarlskog parameter is enhanced by a factor of two. The other elements of the CKM matrix and the angles of the unitarity triangle receive only small corrections, down by a factor  $\tan\beta$  or suppressed by the generation hierarchy present in Yukawa, CKM or  $\mathbf{\Gamma}$  matrices.

## 5 Appendix

Conventions of the Standard Model are fixed by  $\mathcal{L}_{Yukawa} = H_q \bar{Q}_L \lambda_q q_R$ , quark mass matrix rotations by  $m_q^{Diag} = V_q^L m_q V_q^{R\dagger}$  and the CKM matrix is defined as  $V_{CKM} = V_u^L V_d^{L\dagger}$ . In the MSSM the relevant term in the superpotential is then  $W = \hat{\bar{q}} \lambda_q^\dagger \hat{Q} \hat{H}_q$ .

Looking closely at the SUSY threshold corrections to the  $d$  quark masses there are the following 1-loop diagrams contributing significantly in large  $\tan\beta$  limit.

*i) gluino diagram*

Using Dirac notation the quark-squark-gluino interaction, relevant for this paper, reads

$$\mathcal{L}_{int} = -\sqrt{2}g_3 \left(\frac{\lambda^A}{2}\right)_{ab} \{ (\bar{d}_a P_R \tilde{g}^A) \tilde{d}_{Lb} - \tilde{d}_{Ra}^\dagger (\tilde{g}^A P_R d_b) \} + h.c.. \quad (29)$$

The *squark* interaction eigenstates are turned into the mass eigenstates according to

$$\tilde{d}_{Ri} = (V_d^{R0\dagger})_{ij} (\Gamma_{dR}^\dagger)_{j\alpha} \tilde{d}_\alpha \quad (30)$$

$$\tilde{d}_{Li} = (V_d^{L0\dagger})_{ij} (\Gamma_{dL}^\dagger)_{j\alpha} \tilde{d}_\alpha. \quad (31)$$

As indicated in these equations, the  $V$  matrices rotate squarks the same way as they do with quarks. The additional rotations are then performed by the 6x3 matrices  $\Gamma_{dL,R}$ . Rules for the Feynman diagrams using this notation can be found in [11]. Note that the indices  $i, j, \dots$  denote generation indices 1,2,3, the greek letters denote squark indices 1 to 6 and that the implicit summation over the repeating indices is assumed. Diagram with the gluino and  $d$ -type squarks in the loop contributes to the *quark* self-energy matrix (amputated two-point function) as :

$$\begin{aligned} -i\Sigma &= (-i\sqrt{2}g_3)^2 C_2(\mathbf{3}) \int \frac{d^d k}{(2\pi)^d} (-) P_R < \tilde{g} \tilde{g} > P_R \\ &V_d^{L0\dagger} \Gamma_{dL}^\dagger < \tilde{d} \tilde{d}^\dagger > \Gamma_{dR} V_d^{R0} + P_L \dots P_L + \not{p} - terms. \end{aligned} \quad (32)$$

Only the term with the two right-handed projectors corrects the mass matrix. The term indicated as  $P_L \dots P_L$  contains similar corrections to  $\mathbf{m}^\dagger$ . To get to the formula (1) in the text one performs the rotation to Euclidean space and

integrates out angular variables. The integral measure  $dk$  in (2) stands for  $k^2 d(k^2)$  and the integration limits are assumed to be zero and infinity. Note that in the main text  $\mathbf{m}_d^0$  was appended to these equations in not a very elegant way, but that is for later convenience.

ii) *chargino diagram*

Quark-squark-chargino interaction that is relevant for this paper reads

$$\begin{aligned} \mathcal{L}_{int} = & (\bar{d} P_R (V^{*\dagger})_{2A} \tilde{\chi}_A^c) \lambda_u \tilde{u}_R + \tilde{u}_L^\dagger \lambda_d ((U^{*\dagger})_{2A} \tilde{\chi}_A^c P_R d) \\ & - g_2 (\bar{d} P_R (V^{*\dagger})_{1A} \tilde{\chi}_A^c) \tilde{u}_L + h.c. . \end{aligned} \quad (33)$$

u squarks are rotated to their mass eigenstates in exactly the same way as the d squarks above, defining the  $\Gamma_{uR,L}$  matrices. Contribution to the d quark self-energy from this interaction reads

$$\begin{aligned} -i\Sigma = & (i)^2 \int \frac{d^d k}{(2\pi)^d} U_{A2} V_{B2} P_R \langle \tilde{\chi}_A^c \tilde{\chi}_B^c \rangle P_R \lambda_u V_u^{R0\dagger} \Gamma_{uR}^\dagger \langle \tilde{u} \tilde{u}^\dagger \rangle \Gamma_{uL} V_u^{L0} \lambda_d - \\ & - U_{A2} V_{B1} P_R \langle \tilde{\chi}_A^c \tilde{\chi}_B^c \rangle P_R g_2 V_u^{L0\dagger} \Gamma_{uL}^\dagger \langle \tilde{u} \tilde{u}^\dagger \rangle \Gamma_{uL} V_u^{L0} \lambda_d + P_L \dots P_L + \not{p} terms . \end{aligned}$$

$U$  and  $V$  diagonalize the chargino mass matrix. The fact that one of their indices is 1 (2), traces back the wino (higgsino) interaction in the quark-squark-chargino vertex of the loop. Summation over  $A, B=1,2$  is assumed. Explicit forms of the  $U$  and  $V$  matrices and further details about the notation can be found in ref.[9]. In order to derive the equation (1) one has to use the relations between the diagonalized and non-diagonalized mass and  $\lambda$  matrices, briefly mentioned at the beginning of this appendix. The vev of the scalar Higgs  $H_d$  is added in order to pull out the mass matrix on the r.h.s. for future convenience and when combined with  $\tan\beta$  (which is pulled out into the  $\epsilon$ ) it yields  $(v_u)^{-1}$  in the final expression (3) given in the text.

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